Piketty's *Capital in the Twenty-First Century* - A Critique[1]

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[1] This paper is an extended version of Rowthorn (2014). This version includes a more extensive treatment of Pikettty's long-run dynamics and a section on the implications of savings out of wages.

Thomas Piketty's *Capital in the Twenty-First Century* (2014) documents long-term trends in wealth ownership and income distribution in advanced economies. It shows how the share of income accruing to wealth-owners has risen in many countries in recent decades[2] . It also provides a simple explanation of this development based on the standard neoclassical theory of factor shares. This theory establishes a link between the capital intensity of production and the share of profits in total output. The nature of this link depends on the elasticity of substitution between capital and labour. When this elasticity is greater than unity, an increase in the capital-output ratio leads to an increase in the share of profits. This, in essence, is Piketty's explanation for the increased share of wealth-owners in national income.

The above explanation has two related flaws. Piketty's assumption regarding the elasticity of substitution is not correct. There is considerable evidence that this elasticity is less than unity. Moreover, Piketty's method for measuring changes in the capital-output ratio is misleading. He fails to allow for the disproportionate increase in the market value of certain assets, especially housing, in recent decades. This leads him to conclude, mistakenly, that the capital-output ratio has risen by a considerable amount. In fact, conventional measures of this ratio indicate that it has been either stationary or has fallen in most advanced economies during the period in question. This would suggest that the basic problem has not been the over-accumulation of capital, but just the opposite. There has been too little real investment.

Piketty also discusses future trends. He lays particular stress on the gap between the rate of return on capital and the growth rate of the economy. He argues that this gap ($r-g$ in his notation) will get wider in the future because the long-term growth rate g will decline as population growth and technical progress decelerate[3]. He argues that this development will lead to an increasing concentration of wealth ownership and the emergence of a rentier class which lives mainly or entirely on the income from investments. In this context Piketty considers the behaviour of the ratio s/g, where s is the average propensity to save. As a matter of arithmetic, s/g is equal to the overall ratio of wealth to income in the society. If this ratio increases, as Piketty expects, the greater will be the importance of inherited wealth in

[2] See Piketty (2014), Figure 6.5 on page 222.

[3] See especially Piketty (2014) pp. 353 to 358.

the life chances of future individuals and hence the greater the degree of inherited inequality. Piketty's discussion of future trends in the inequality of capital ownership and its implications for inheritance is both interesting and plausible. However, there is one important omission. There is little discussion of future trends in factor shares. What, for example, is the implication of a lower economic growth rate for the share of profits in national income? This issue is discussed below. It turns out that, as in the historical analysis, the elasticity of substitution plays a central role.

A word of warning is in order. The following analysis is based on the neo-classical theory of factor shares. This theory is often condemned on both theoretical and empirical grounds, but it is the theory which Piketty uses and I am merely following his example. Moreover, despite its weaknesses, I believe that this theory throws some light on reality.

The Determination of Factor Shares

This paper uses a simple model to explore Piketty's analysis of income dynamics. Apart from slight notational differences, this model is similar to that described by Piketty in his technical appendix[4]. The present model also makes an explicit allowance for variations in the market valuation of real assets and for technical progress. Technical progress is of the labour-augmenting (Harrod-neutral) variety. This choice of technical progress is motivated by the desire to analyse balanced growth paths in which capital and output grow at the same rate. It is also supported by the evidence (Klump et al, 2007). In the economies that Piketty considers, net income from abroad has for most of the time been a small fraction of total income. Net income from abroad is assumed to be zero in our model.

Preliminary Remarks

Piketty uses the terms "capital" and "wealth" interchangeably to denote the total monetary value of shares, housing and other assets. "Income" is measured in money terms. We shall reserve the term "capital" for the totality of productive assets evaluated at constant prices. The term "output" is used to denote the totality of net output (value-added) measured at constant prices. Piketty uses the symbol β to denote the ratio of "wealth" to "income" and he denotes the share of wealth-owners in total income by α. In his theoretical analysis this share is equated to the share of profits in total output. Piketty documents how α and β have both risen by a considerable amount in recent decades. He argues that this is not mere correlation,

[4] See http://piketty.pse.ens.fr/files/capital21c/en/Piketty2014TechnicalAppendix.pdf, pages 37-39.

but reflects a causal link. It is the rise in β which is responsible for the rise in α. To reach this conclusion, he first assumes that β is equal to the capital-output ratio K/Y, as conventionally understood. From his empirical finding that β has risen, he concludes that K/Y has also risen by a similar amount. According to the neoclassical theory of factor shares, an increase in K/Y will only lead to an increase in α when the elasticity of substitution between capital and labour σ is greater than unity. Piketty assumes this to be the case. Indeed, based on movements α and β, he estimates that σ is between 1.3 and 1.6[5]

Thus, Piketty's argument rests on two crucial assumptions: β = K/Y and σ > 1. Once these assumptions are granted, the neoclassical theory of factor shares ensures that an increase in β will lead to an increase in α. In fact, neither of these assumptions is supported by the empirical evidence which is surveyed briefly in the appendix. This evidence implies that the large observed rise in β in recent decades is not the result of a big rise in K/Y but is primarily a valuation effect.

The Model

Real output is given by the following CES production function:

(1) $$Y = \left[bK^{\frac{\sigma-1}{\sigma}} + (1-b)\left(Le^{\mu t}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where μ is the constant rate of labour-augmenting technical progress and σ > 0 is the constant elasticity of substitution between capital K and labour L. The parameter b is constant.

Following Piketty, assume that capital receives its marginal product. The rate of profit is thus:

(2) $$\pi = \frac{\partial Y}{\partial K}$$

which yields:

(3) $$\pi = b\left(\frac{K}{Y}\right)^{-\frac{1}{\sigma}}$$

[5] Piketty (2014), chapter 6, page 221. Also, the online technical appendix page 39.

The share of profits in output is given by:

(4) $$\alpha = \frac{\pi K}{Y} = b\left(\frac{K}{Y}\right)^{\frac{\sigma-1}{\sigma}}$$

Growth rates of the above variables satisfy the following equation:

(5) $$g_{\alpha} = \left(\frac{\sigma-1}{\sigma}\right) g_{K/Y}$$

Thus, g_{α} and $g_{K/Y}$ have the same sign if σ > 1 and opposite signs if σ < 1. This is a standard neoclassical result.

Piketty

Piketty does not measure K/Y directly but takes as a proxy the ratio of wealth to income, where wealth is the total monetary value of shares, housing and other assets; income is measured in money terms. The inclusion of housing is questionable, since housing is not combined with labour in a production process in the same way as other types of capital. There is also the question of valuation. Taking produced goods as numeraire, let W be the market value of capital (stocks and shares, housing etc.) and define the valuation ratio as follows:

(6) $$v = \frac{W}{K}$$

In the case of quoted companies this is Tobin's Q.

Piketty's wealth to income ratio is given by:

(7) $$\beta = \frac{W}{Y} = \frac{vK}{Y}$$

Growth rates of the above variables satisfy the following equation:

(8) $$g_{\beta} = g_{K/Y} + g_{v}$$

In his explanation for the changing distribution of income Piketty finesses the issue of valuation by assuming, in effect, that $g_v = 0$ and hence that $g_{K/Y} = g_{\beta}$. Given his finding that β has increased by a great deal in recent decades, Piketty concludes that K/Y must have increased by a similar amount. However, evidence surveyed in the appendix indicates that K/Y has been falling since around 1981-2 in the United States and has been roughly constant in most of Europe. Indeed, this is just what Piketty and Zucman (2013) find when they correct the wealth-income ratio for valuation changes (capital gains)[6]. Piketty's tacit assumption that $g_v = 0$ is also at odds with his own evidence which documents the increase in the average valuation ratio of quoted companies (Tobin's Q) that has occurred since 1970[7].

The following is a plausible story, at least for the United States, where the capital-output ratio has fallen a great deal and thus $g_{K/Y} < 0$. Evidence reported in Appendix 1 indicates that σ < 1. Suppose this is the case. Suppose also that $g_v > -g_{K/Y}$. Then, in line with Piketty's empirical findings, $g_\beta = g_{K/Y} + g_v > 0$. Given that σ < 1 and $g_{K/Y} < 0$, the neoclassical theory of factor shares implies that $g_\alpha > 0$, which is also in line with Piketty's empirical finding. In this example, the income share of wealth-owners is increasing because of a low rate of real capital formation and a falling capital-output ratio. However, the wealth-income ratio is increasing because of a rapid growth in asset prices. This story, it must be said, assumes that the neoclassical theory is correct. In particular, it assumes that capital receives its marginal product. If this assumption is incorrect, a different or more complex explanation for the rising income share of wealth-owners is required. Such an explanation might include the declining economic and political power of organised labour in most advanced economies. However, this would not preclude low real investment as a contributory factor behind the observed shift in income distribution.

Balanced Growth

Assume that a constant fraction s of income is saved. The growth rate of the real capital stock is then given by:

(9) $$g_K = \frac{sY}{K}$$

Suppose that employment grows at the exogenous rate ℓ. With the assumed savings propensity the economy will converge to a balanced growth path on which capital and output grow at the same rate g, where:

[6] Piketty and Zucman (2013), appendix figure A133, available on-line at http://piketty.pse.ens.fr/en/capitalisback
[7] See Piketty(2014), chapter 6, Figure 5.7 and his discussion on pages 187-191.

(10) $g = \ell + \mu$

In his book, Piketty refers to g as the "structural growth rate of the economy", although elsewhere he uses the conventional term "natural growth rate" (Piketty and Zuckman, 2013, p.6).

Thus, on the balanced growth path:

(11) $g_K = g_Y = g$

The capital-output rate is:

(12) $\frac{K}{Y} = \frac{s}{g}$

Thus:

(13) $\pi = \left(\frac{s}{g}\right)^{\frac{1}{\sigma}}$

(14) $\pi - g = \left(\frac{s}{g}\right)^{\frac{1}{\sigma}} - g$

(15) $\frac{\pi K}{Y} = b\left(\frac{s}{g}\right)^{\frac{\sigma-1}{\sigma}}$

Piketty lays considerable stress on the ratio s/g and the gap $\pi - g$ ($r - g$ in his notation). He argues that both s/g and $\pi - g$ will rise in the future due to a reduction in the natural growth rate g and possibly an increase in the savings propensity *s* due to the increasing concentration of wealth. The effect of this on the distribution of factor income depends on the elasticity of substitution σ. As can be seen from equation (15), the share of profits is an increasing function of s/g if σ >1 and a decreasing function if σ < 1. This is an important finding. Piketty assumes that σ >1. In this assumption is correct, a reduction in the natural growth rate g (or increase in s) will lead to a new balanced growth path on which the share of profits is higher than before. Conversely, if σ < 1, the share of profits will be lower than before.

An Alternative Savings Function

Piketty is concerned about the emergence of a class of rentiers who do not work and live on the income from their investments. The above model can be easily modified to accommodate such a development. Suppose there are two social classes: workers and capitalists. The latter do not work and live entirely on their income from profits. Workers save a constant fraction s_w of their income and capitalists save a constant fraction s_c. Workers and capitalists receive rates of return π_w and π_c respectively, where

(16) $$\pi_w = m\pi_c$$

It is assumed that $s_c > s_w$ and $m \leq 1$ is constant. There is no capital appreciation. Output is determined by the production function given in equation (1). On average factors receive their marginal products, so the average rate of profit π is given by equation (3).

Using an obvious notation, the profits of workers and capitalists are given by:

(17) $$\begin{aligned}\Pi_w &= \pi_w K_w \\ \Pi_c &= \pi_c K_c\end{aligned}$$

Aggregates are given by:

(18) $$\begin{aligned}\Pi &= \Pi_w + \Pi_c \\ K &= K_w + K_c\end{aligned}$$

The total income of workers is equal to $Y - \Pi_c$ and the savings of each group are as follows:

(19) $$\begin{aligned}S_w &= s_w(Y - \Pi_c) \\ S_c &= s_c\Pi_c\end{aligned}$$

Suppose the economy is on a balanced growth path along which output and the capital stock owned by each class grow at the natural rate $g = \ell + \mu$. Thus:

(20) $$g_{K_w} = g_{K_c} = g_K = g_Y = g$$

It is shown in appendix 2 that the average rate of profit π on this balanced growth path is the unique solution to the following equation:

(21) $$s_w s_c(1-m)\left(\frac{\pi}{b}\right)^{\sigma}+(s_c-s_w)s_c\pi-\left(s_c-s_w m\right)g=0$$

If $m=1$ the solution is $\pi=\dfrac{g}{s_c}$. Total savings are then equal to $S=gK=s_c\pi K=s_c\Pi$. This result was first established by Pasinetti (1962).

As always the capital-output ratio is:

(22) $$\frac{K}{Y}=\left(\frac{\pi}{b}\right)^{-\sigma}$$

The profit share is:

(23) $$\frac{\pi K}{Y}=b^{\sigma}\pi^{1-\sigma}$$

For $m<1$ equation (21) implies that:

(24) $$\begin{array}{l} \partial\pi/\partial s_c<0 \text{ and } \partial\pi/\partial g>0 \\ \partial\pi/\partial s_w<0 \text{ for } m<1,\ =0 \text{ for } m=1 \end{array}.$$

Thus, an increase in the savings propensity for either class or a reduction in the growth rate will normally reduce the average profit rate. The only exception occurs when workers and capitalists enjoy the same rate of return on their investments, in which case variations in s_w have no effect of the profit rate. The effect of such changes on the profit share can be inferred from equation (23). If $\sigma<1$ the profit share is normally an increasing function of s_w and s_c and is a decreasing function of g. The reverse is the case if $\sigma>1$. Thus, changing the savings function does not alter the qualitative results regarding the capital-output ratio, the rate of profit or the profit share.

Capital Appreciation

The consumption of wealth-owners may be influenced by the gains they make from capital appreciation. For example, a shareholder may increase his consumption out of dividends income because of the capital gain he is making from rising share prices. In aggregate, capital gains are assumed to be unrealised. This ensures that the consumption of wealth-owners cannot exceed their current income from profits[8]. When such gains are included, the real rate of return on wealth is given by:

$$(25) \qquad r = \frac{\pi K + \frac{dv}{dt} K}{vK}$$

Hence:

$$(26) \qquad r = \frac{\pi}{v} + g_v$$

where g_v is the growth rate of v.

Suppose that workers do not save and that wealth-owners consume a constant fraction $1 - s_r$ of their real income including capital appreciation. Consumption by wealth-owners is then equal to $(1 - s_r) rW = (1 - s_r) rvK$. Subtracting this amount from the real profit flow πK yields the following expression for the real savings to be invested in additional capital stock:

$$(27) \qquad \begin{aligned} S &= \pi K - (1 - s_r) rvK \\ &= \pi K - (1 - s_r)\left(\frac{\pi}{v} + g_v\right) vK \\ &= \left(s_r \pi - (1 - s_r) g_v v\right) K \end{aligned}$$

Dividing by K yields:

$$(28) \qquad g_K = s_r \pi - (1 - s_r) g_v v$$

The assumption that in aggregate capital gains are not realised ensures that $S \geq 0$ and hence $g_K \geq 0$. If g_v is positive, the final term indicates that real investment is reduced because

[8] Wealth-owners could in aggregate realise some of their capital gains by selling some of their capital assets to workers. This possibility is ruled-out by our assumption that workers do not save (see below).

wealth-owners are consuming more as a result of capital appreciation. Their real wealth is increasing because of rising asset prices and they have less need to save out of their current income (profits).

Suppose that employment grows at the exogenous rate ℓ. The growth rate of output is then:

(29) $$g_Y = \alpha g_K + (1-\alpha)g$$

where $g = \ell + \mu$ is the natural growth rate. The capital-output ratio grows at the following rate:

(30) $$g_{K/Y} = (1-\alpha)(g_K - g)$$

This ratio will fall through time if $g_K < g$. Equation (28) implies that this will occur when:

(31) $$s_r\pi - g < (1-s_r)g_v v$$

Piketty's wealth-income ratio is β = W/Y = vK/Y which has growth rate:

(32) $$\begin{aligned} g_\beta &= g_{K/Y} + g_v \\ &= (1-\alpha)(s_r\pi - (1-s_r)g_v v - g) + g_v \\ &= (1-\alpha)(s_r\pi - g) + \left[1-(1-\alpha)(1-s_r)v\right]g_v \end{aligned}$$

The above growth rate is positive if:

(33) $$s_r\pi - g > (1-s_r)vg_v - \frac{g_v}{1-\alpha}$$

The condition for the two inequalities $g_{K/Y} < 0$ and $g_\beta > 0$ to hold simultaneously is

(34) $$(1-s_r)vg_v - \frac{g_v}{1-\alpha} < s_r\pi - g < (1-s_r)vg_v$$

As always:

(35) $$g_\alpha = \left(\frac{\sigma-1}{\sigma}\right)g_{K/Y}$$

If σ < 1 and the inequalities (34) are satisfied, the capital-output ratio will fall in the course of time and the share of wealth-owners in total income will rise. However, the capital-output ratio will fall because there is so little real investment. Because of capital appreciation, wealth-owners are able to enjoy a high level of consumption and at the same time see their wealth growing faster than total income. This is a fair description of what has happened in a number of countries.

Conclusions

Piketty argues that the higher income share of wealth-owners is due to an increase in the capital-output ratio resulting from a high rate of capital accumulation. The evidence suggests just the contrary. The capital-output ratio, as conventionally measured, has either fallen or remained constant in recent decades. The apparent increase in the capital-output ratio identified by Piketty is a valuation effect reflecting a disproportionate increase in the market value of certain assets. A more plausible explanation for the increased income share of wealth-owners is an unduly low rate of investment in real capital. These alternative explanations have distinct policy implications which it is beyond the scope of this paper to explore.

Piketty makes a great deal of the gap $\pi - g$ and the ratio s/g. He argues that g is likely to fall in the future because of decelerating population and productivity growth. The result will be an increase in both $\pi - g$ and s/g. He concludes that these changes will be accompanied by an increase in the wealth to income ratio (capital-output ratio). This in turn will encourage the concentration of wealth and promote the rise of a rentier class living on inherited wealth. These conclusions are probably correct. However, this leaves open the question of factor shares. The share of profits in total income may rise or fall depending on the elasticity of substitution between capital and labour. If $\sigma < 1$ then the profit share will fall, despite a rising ratio of wealth to income.

Appendix 1: Evidence

Valuation

For any group of assets define the valuation ratio v as follows:

$$v = \frac{\text{total market value of assets}}{\text{total replacement cost of assets}}$$

In the case of quoted companies, this ratio is usually known as Tobin's Q, although in fact the term valuation ratio was earlier coined by Marris (1964).

Suppose that "real" capital K is measured in such a way that, for the whole economy or for the private sector as a whole, its unit replacement cost is on average equal to the unit price of real output Y. Then:

$$\beta = \frac{W}{Y} = \frac{vK}{Y}$$

In assuming that β = K/Y, Piketty is, in effect, assuming that v = 1. Casual observation suggests that this is not true for house prices, which have risen much faster than building costs in many countries due to rising land values. Using data from Canada, France, the United States and the United Kingdom, Bonnet et al (2014) show that the increase in identified by Piketty was mainly due to the rapid growth in house prices. Preliminary estimates by Bill Martin confirm this finding for the UK[9]. Note that when Piketty and Zucman simulate the effect of excluding capital gains, they find that the adjusted private wealth-income ratio for the United States falls almost continuously over the period 1982 - 2010 and remained virtually flat for a weighted group of European counties[10]. Estimates by Klump et al (2014) of the conventional capital-output ratio for these countries reveal a similar picture. Estimates by Thwaites (2014) of the real capital to gross value-added ratio for an average of 11 industrialised countries indicate that this ratio fell up to the mid-nineteen seventies and since then has been more or less flat.

[9] Piketty estimates that the UK ratio of private wealth to income rose by 69% between 1970 and 2010 (on-line technical appendix table S3.1). When adjusted for changes in the relative price of housing, the increase was 24% (personal communication from Bill Martin to the author).

[10] Piketty and Zucman (2013), appendix figure A133, available on-line at http://piketty.pse.ens.fr/en/capitalisback

The combination of a rising β and a falling or stationary K/Y implies that the valuation ratio v must have been increasing.

Elasticity of Substitution

On page 221, Piketty claims that from historical data one can infer that σ lies between 1.3 and 1.6. However, this inference is unreliable because it is based on the assumption that K/Y = β. Conventional measures of K/Y typically yield values of σ that are much lower than 1.

Rowthorn (1996) and Rowthorn (1999) report the results of 33 econometric studies which estimate the value of σ, or from which estimates of this parameter can be derived. Most of these studies contain a variety of estimates referring to different industries, regions or countries, or to alternative equation specifications. Their findings are summarised by means of employment-weighted averages or medians. Out of a total of 33 studies, in only 7 cases does the summary value exceed 0.8, and the overall median of the summary values (median of the medians) is equal to 0.58. A more recent survey by Klump et al (2007) reports similar findings for aggregate elasticities. These authors' own estimates for the private non-housing sector imply elasticities in the range 0.60-0.67 in both the United States and the Eurozone. A survey by Chirinko (2008) concludes “While some estimates of σ are above one, the weight of the evidence suggests that σ lies in the range between 0.40 and 0.60”. Allowing for biased technical change, Antràs (2004, p. 26) concludes that for the United States σ "is likely to be considerably below one, and may even be lower than 0.5". Allowing for mark-up pricing, Raurich et al (2011) estimate an elasticity of 0.63 for the United States. Using firm-level data, Barnes et al (2008) find a long-run elasticity of 0.4 for the UK. Using data for New Zealand, Tipper (2012) obtains mostly low elasticities for individual industries and estimates the aggregate elasticity to be in the region of 0.8.The latter estimate is not statistically significantly different from unity. Using data for the United States, Balistreri et al (2002) find a wide variety of long run elasticities of substitution at the industry level, with a median of around 1. Allowing for changes in the relative price of investment goods, Karabarbounis, L. and B. Neiman (2014) estimate the elasticity of substitution in a large sample of countries to be in the region of 1.26 .

Appendix 2: Derivations

Equation (21)

On the balanced growth path (A1)

$$\begin{aligned} g_{K_w} = \frac{S_w}{K_w} &= s_w\left(\frac{(Y-\Pi)+\Pi_w}{K_w}\right) \\ &= s_w\left[\left(\frac{Y-\Pi}{K}\right)\frac{K}{K_w}+\frac{\Pi_w}{K_w}\right] \\ &= s_w\left[\left(\frac{Y-\Pi}{K}\right)\frac{K}{K_w}+\pi_w\right] \\ &= s_w\left[\left(\frac{Y}{K}-\pi\right)\frac{K}{K_w}+m\pi_c\right] \end{aligned}$$

Using equations (3), (16) and (20) this can be written as follows:

(A2) $$g = s_w\left[\left(\left(\frac{\pi}{b}\right)^{\sigma}-\pi\right)\frac{K}{K_w}+m\pi_c\right]$$

From the definition of π:

$$\begin{aligned} \pi &= \pi_w\left(\frac{K_w}{K}\right)+\pi_c\left(1-\frac{K_w}{K}\right) \\ &= \pi_c-(\pi_c-\pi_w)\left(\frac{K_w}{K}\right) \qquad \text{(A3)} \\ &= \pi_c\left(1-(1-m)\left(\frac{K_w}{K}\right)\right) \end{aligned}$$

Also:

(A4) $$g = g_{K_c} = s_c\pi_c$$

Thus,

(A5) $$\pi = \frac{g}{s_c}\left(1-(1-m)\left(\frac{K_w}{K}\right)\right)$$

From which it follows that:

(A6) $$\frac{K_w}{K} = \frac{(g-s_c\pi)}{g(1-m)}$$

Substituting in (A3) yields:

(A6) $$g = s_w\left[\left(\left(\frac{\pi}{b}\right)^{\sigma} - \pi\right)\frac{g(1-m)}{\left(g - s_c\pi\right)} + \frac{mg}{s_c}\right]$$

Cancelling g, multiplying by $g - s_c\pi$ and s_c rearranging yields:

(A7) $$s_w s_c(1-m)\left(\frac{\pi}{b}\right)^{\sigma} + (s_c - s_w)s_c\pi - \left(s_c - s_w m\right)g = 0$$

which is the required equation.

Differentials

Write equation (A6) as an implicit function of four variables as follows:

(A8) $$F(\pi, s_w, s_c, g) = s_w s_c(1-m)\left(\frac{\pi}{b}\right)^{\sigma} + (s_c - s_w)s_c\pi - \left(s_c - s_w m\right)g = 0$$

Differentiating partially yields:

(A9) $$\frac{\partial F}{\partial \pi} = s_w s_c(1-m)\frac{\sigma}{b}\left(\frac{\pi}{b}\right)^{\sigma-1} + (s_c - s_w)s_c > 0$$

$$\frac{\partial F}{\partial s_c} = s_w(1-m)\left(\frac{\pi}{b}\right)^{\sigma} + (2s_c - s_w)\pi - g$$

$$= \frac{-(s_c - s_w)s_c\pi + \left(s_c - s_w m\right)g}{s_c} + (2s_c - s_w)\pi - g$$

(A10) $$= s_c\pi - \frac{s_w mg}{s_c}$$

$$= s_c\pi - s_w m\pi_c$$

$$= s_c\pi - s_w\pi_w > 0$$

(A11) $$\frac{\partial F}{\partial g} = -\left(s_c - s_w m\right) < 0$$

$$\frac{\partial F}{\partial s_w} = s_c(1-m)\left(\frac{\pi}{b}\right)^{\sigma} - s_c\pi + mg$$

$$= \frac{-(s_c - s_w)s_c\pi + \left(s_c - s_w m\right)g - s_w s_c\pi + s_w mg}{s_w}$$

(A12) $$= \frac{s_c\left(g - s_c\pi\right)}{s_w}$$

$$= \frac{s^2{}_c\left(\dfrac{g}{s_c} - \pi\right)}{s_w}$$

$$= \frac{s^2{}_c\left(\pi_c - \pi\right)}{s_w} > 0 \text{ if } m < 1,\ 0 \text{ if } m = 0.$$

$$\frac{\partial \pi}{\partial s_c} = -\frac{\dfrac{\partial F}{\partial s_c}}{\dfrac{\partial F}{\partial \pi}} < 0, \frac{\partial \pi}{\partial g} = -\frac{\dfrac{\partial F}{\partial g}}{\dfrac{\partial F}{\partial \pi}} > 0$$

$$\frac{\partial \pi}{\partial s_w} = -\frac{\dfrac{\partial F}{\partial s_w}}{\dfrac{\partial F}{\partial \pi}} < 0 \text{ if } m < 1,\ 0 \text{ if } m = 1.$$

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